Everest v. Kilimanjaro: An EER Teaching Demo

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Part I: Misconception

Given *no information except what you already know about the Earth*, what would you estimate the following to be? [*This question is intended for students, but for this demo, you are the students.*]

Estimate the value of $h_E - h_K$, where h_E is the distance from the center of the Earth to the top of Mt. Everest, and h_K is the distance from the center of the earth to the top of Mt. Kilimanjaro?

If you have enough Earth knowledge to jump bravely into this estimation problem, it is likely that you are assuming that the Earth is a sphere and therefore the distance from the center of the Earth to sea level is the same for both mountains (the radius of the sphere); hence, the answer to the question would be the difference between the elevations of the two mountains. Some of you will know that Everest is about 29,000 feet above MSL, and a few might know that Kilimanjaro rises to about 19,000 feet. This thinking leads to an estimated value of about 10,000 feet in favor of Everest as the summit farther of the two from the center of the Earth. (Working together in groups might help with the actual numbers.)

But you (and the groups) would be wrong! Why? Because the Earth is not a sphere! When Isaac Newton was discussing how the planet spun on its axis, he stated that to some amount, the planet should bulge near the Equator because of the rotation. He was correct, of course. The Earth is actually an oblate spheroid (ellipsoid of revolution). Think of a rigid exercise ball that Archimedes sat on, causing a slight flattening at the poles and a bulging in the middle.

Part II: Estimation (Back of the envelope)

Because the Earth is an *oblate* (as opposed to *prolate*) spheroid (i.e., the equatorial radius is larger than the polar radius), the distance from the center of the Earth to the top of Mt. Everest is actually smaller than the distance from the center of the earth to the top of Mt. Kilimanjaro (which you may have anticipated, because why else would we ask the question?)

Question: Given that the elevation and latitude of Mt. Everest are about 29,000 ft and 28 degrees N, respectively, and the elevation and latitude of Mt. Kilimanjaro are about 19,000 ft and 3 degrees S, respectively, **estimate** the amount that the distance from the center of the earth to the top of Mt. Kilimanjaro exceeds the distance from the center of the earth to the top of Mt. Everest.

An answer. The difference can be estimated as follows.

1. The difference in elevation is about 10,000 ft. The problem then reduces to how much farther than 10,000 ft is sea level at the latitude of Mt. Kilimanjaro from the center of the earth than sea level at the latitude of Mt. Everest?

2. For our estimate, **assume** that Mt. Kilimanjaro is at 0 degrees latitude and that Mt. Everest is at 30 degrees latitude. (Very convenient numbers, as you will see.)

3. **Recall** that the Earth is approximately a sphere with circumference of 40,000 km (why?). Then, the implied radius is 6366 km (Why?).

4. **Recall** that the flattening of the Earth, $f = \frac{a-b}{a}$, where a is the semimajor axis (equatorial radius) and the b is the semiminor axis (polar radius), is 1/298 (or 0.0335%). Then, the equatorial radius is 21 km longer than the

polar radius. “Splitting the difference” with 6366 km in the middle, we **estimate** that $a = 6376.5$ km and $b = 6355.5$ km.

5. Now**, recall**: the parametric equations of an ellipse are $X(t) = a\cos(t)$ and $Y(t) = b\sin(t)$, where t is an angle. Actually, the angle, called the *eccentric anomaly*, is difficult to describe verbally, but see https://en.wikipedia.org/wiki/Ellipse#/media/File:Parametric_ellipse.gif. As shown in the gif, t is not the same as the latitude (the angle, θ, between the radius, r, from the center to the point (x, y) on the spheroid's cross-sectional ellipse), but the two are close (i.e., $\theta \approx t$, especially as the difference between the circle of radius a and the circle with radius b diminishes; see gif). In general, $\tan t = \frac{a}{b}\tan\theta$, (note the similarity between this equation and the equation for comparing angles with vertical exaggeration; here the cross-sectional ellipse of an oblate spheroid can be viewed as a case of horizontal exaggeration, in which the ratio, $\frac{a}{b}$, becomes the horizontal exaggeration. In the case of the a and b of this problem, the horizontal exaggeration is 1.0033). So, we can **simplify** and use latitude in the parametric equations, and then move on to calculate the (x, y) coordinates of sea level at the locations of Mt. Kilamanjaro and Mt. Everest.

6. For the (x, y) coordinates of sea level at Mt. Everest, **recall** the ever-useful 30-60-90 triangle. Here, the 30 degrees corresponds to the latitude. The x is the distance from the center of the earth, along an equatorial radius to a point exactly below the point on the ellipsoid at Mt. Everest. The y is the vertical distance from the equatorial plane to the point on the ellipsoid at Mt. Everest (it might be helpful if you made a sketch on the back of that envelope). The x-value is given by $a\cos(30°)$, and the y-value is given by $b\sin(30°)$. **Recalling** that the lengths of a 30-60-90 triangle are in the ratio of $1:\sqrt{3}:2$, then $\cos(30°) = \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.866$, and $\sin(30°) = \frac{1}{2} = 0.500$. And so, we have $x = (6376.5\text{ km})(0.866) = 5522.2\text{ km}$, and $y = (6355.5\text{ km})(0.500) = 3177.75\text{ km}$. Finally, from Pythagoras, we have the distance from the origin (center of the Earth) to the (x, y) on our spheroid at Mt. Everest to be $r = \sqrt{x^2 + y^2} = \sqrt{5522.2^2 + 3177.75^2} = 6371.3\text{ km}$.

7. The (x, y) coordinates of sea level at Mt. Kilimanjaro are even easier because we are assuming that the latitude is zero. Thus $x = a = 6376.5\ km$, and $y = 0$, and so $r = \sqrt{x^2 + y^2} = a = 6376.5\text{ km}$.

8. **Thus we estimate the distance from the center of the earth to sea level at Mt Kilimanjaro to be 5.2 km longer than it is to sea level at Mt Everest. This difference amounts to about 17,000 ft. It exceeds the difference in altitudes of 10,000 ft by 7,000 ft. Thus we estimate Mt. Kilimanjaro to be about 7000 ft (1.3 mi) further away from the center of the Earth than Mt. Everest is.**

Part III: Reality

As noted here, the calculation we've shown is an estimate. In reality, the shape of the Earth (geoid) is more complex even before the irregularities of the surface (physiography) are taken into account. Visit http://mathscinotes.com/2015/01/the-farthest-mountaintops-from-the-center-of-the-earth/ to see the more detailed calculations – this more detailed set of calculations can also be a teaching and learning opportunity, especially if you use computers in your classroom.

As noted in the given weblink, the actual difference is about 1.7 km, which is about 5600 ft. Our estimate is off by about 1400 feet; that's about one lap around an Olympic track.

See http://vicricchezza.weebly.com/publications.html for a copy of this handout and other (hopefully) interesting items!