Unit Sticks: An EER Teaching Demo

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Pre-lab Assessment

Evaluate and solve the following terminated continued fraction:

$$3 + \cfrac{1}{2 + \frac{1}{3}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Show all work, give answer as reduced fraction – hint: start at the bottom – no calculators!
Solution:

Please read this after completing the arithmetic

If you're stumped about where this situation could ever actually happen, here's some information from Joseph Louis Lagrange (remember him?) that answers the question on how to do this and why it's useful:

> *Suppose, for example, that you have a given length, and that you wish to measure it. The unit of measure is given, and you wish to know how many times it is contained in the length. You first lay out the measure as many times as you can on the given length, and that gives you a certain whole number of measures. If there is no remainder, your operation is finished. But if there is a remainder, that remainder is still to be evaluated...*
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> *…If you have a remainder, since that is less than the measure, naturally you will seek to find how many times your remainder is contained in this measure. Let us say two times, and a remainder is still left. Lay this remainder on the preceding remainder. Since it is necessarily smaller, it will still be contained a certain number of times in the preceding remainder, say three times, and there will be another remainder or there will not; and so on. (Lagrange 1795)*

Activity

Note: a peer-reviewed note on the activity in this demonstration was published in Numeracy *earlier this month as "On a Desert Island with Unit Sticks, Continued Fractions and Lagrange" (Ricchezza and Vacher 2016) and is used with permission of the authors. For more information, scan the QR code in the top header, which will take you to Vic's website with much more to see. You can also go directly to http://vicricchezza.weebly.com/publications.html*

Suppose you were stranded on a desert island – most of us here at this conference are old enough to have seen *Gilligan's Island* – and you (as the obvious "Professor" in this scenario) are forced to make a functioning radio out of coconuts and call for help, or some similarly amusing but ridiculous scenario. One problem that becomes obvious immediately is that you don't have a ruler! Cell phones with apps to that purpose are all out of charge. What will you do?

Our late friend Lagrange has already told us the answer. We will select something nearby as our "unit", and will make our measurements from it. Despite the fact that our new unit – we'll call it a stick, because we probably will be able to find a couple of sticks on a desert island – doesn't come with graduations (the convenient little marks that show you where inches or centimeters fall on your normal ruler), we can still make reasonably accurate measurements if we are diligent.

For this activity demonstration, you will measure the width of your table top with an ungraduated stick (in this case, a disposable chopstick survived the crash with you). The procedure, as Lagrange has already shown us, is relatively simple, although it can be somewhat time-consuming. You will use the stick to measure how many whole lengths the table measures; we will call that number s. There will, almost certainly, be some sort of remainder (I [VR] am writing these instructions without access to the demo room, but I'm thinking the chances that the stick will divide evenly into a random table are very small). You will mark the length of the remainder (let's call it r_1) on an unlined piece of paper we'll provide you. Then you'll measure out how many times r_1 fits into the stick, and if there's anything left, it will be called r_2. We then measure r_2 against the length of r_1, and so on. In theory, this routine could go on indefinitely, but given the visual acuity of the human eye, and the inherent measurement error in picking up and laying down a stick repeatedly, going beyond about three remainders is probably pushing things. You then set up your measurements as a continued fraction, and reduce the fraction. You can convert that to a decimal if you wish.

$$s + \cfrac{1}{r_1 + \cfrac{1}{r_2 + \cfrac{1}{r_3}}} = width\ of\ table$$

Note that for a class, this activity would involve both length and width: students would be asked to provide three measurements of each, to calculate the *area* of the table's top surface, and to estimate an error range for the measurements and calculated results.

References Cited

Lagrange, Joseph Louis. 1795. "Lectures on Elementary Mathematics."

Ricchezza, Victor J., and H.L. Vacher. 2016. "On a Desert Island with Unit Sticks, Continued Fractions and Lagrange." *Numeracy* 9 (2). doi: http://dx.doi.org/10.5038/1936-4660.9.2.8.